KIRWAN STATE HIGH SCHOOL	SUBJECT: SPECIALIST MATHEMATICS	UNIT: 3

Unit Description	Unit Objectives
In Unit 3, students will develop the mathematical understandings and skills to solve problems relating to: • Topic 1: Proof by mathematical induction • Topic 2: Vectors and matrices • Topic 3: Complex numbers 2. Proof by mathematical induction continues the developmental concept of proof from Units 1 and 2. Unit 1 introduced a study of vectors with a focus on vectors in two-dimensional space. Unit 2 introduced complex numbers; Unit 3 extends the study of complex numbers to include complex arithmetic using polar form. In this unit, students explore applications of matrices, study three-dimensional vectors, and are introduced to vector equations and vector calculus, with the latter extending students' knowledge of calculus from Mathematical Methods. Cartesian equations and vector equations, together with equations of planes, enable students to solve geometric problems and problems involving motion in three-dimensional space. These topics build on prior knowledge to enable a greater depth of analytical thinking and metacognition.	Students will: 1. select, recall and use facts, rules, definitions and procedures drawn from all Unit 3 topics 2. comprehend mathematical concepts and techniques drawn all Unit 3 topics 3. communicate using mathematical, statistical and everyday language and conventions 4. evaluate the reasonableness of solutions 5. justify procedures and decisions by explaining mathematical reasoning 6. solve problems by applying mathematical concepts and techniques drawn from all Unit 3 topics.

Assessment Plan:

Task	%	Objectives to be assessed	Conditions	Date
IA1 – Internal Assessment 1 PSMT – *based on Unit 3, topic 2*	20	As above – all objectives included on assessment item	4 weeks - including 3 hours of class time	Term 1 Week 5
Task	**%**	**Objectives to be assessed**	**Conditions**	**Date**
IA2 – Internal Assessment 2 Examination – *representatively sample all Unit 3 topics not assessed by the PSMT*	15	As above – all objectives included on assessment item	Closed Book QCAA formula sheet required Technology Free Technology Active 120 minutes + 5 minutes perusal	Term 2 Week 2

Monitoring and Reviewing:

Strategies for Monitoring Student Progress	Date	Planned Reviews at Key Intervals	Date
Student Summary Rule book – separate book following through all units Proficiency scales Regular vocabulary review, HW – weekly review, Formative items Common mistakes recognition Use of online support – Education Perfect, Khan Academy, Text-based online support Graphic organisers – e.g. mind maps, Frayer model, KWL (what I know, what I want to know, what I have learnt)		10 minute review (weekly quiz) during one lesson a week Mathspace quizzes - weekly Formative items	Each week Week 5 Week 10

Underpinning Factors:

Guaranteed Vocabulary:

- domino effect
- element
- generalisation
- inductive step
- initial statement
- proof by mathematical induction
- proposition
- QED - quod erat demonstrandum
- RTP-required to prove
- component form
- concurrent
- cross product
- direction vector
- displacement
- Dominance matrices
- ellipse
- equilibrium
- first-,second-, third-order dominance (etc)
- Gaussian techniques
- hyperbola
- Leslie matrices
- magnitude
- altitude
- angle of projection
- angular speed
- angular velocity
- augmented matrix
- bearing
- Cartesian equation of a line in two- and three-dimensions
- Cartesian equation of a plane
- circular motion
- coefficient matrix
- collinear
- projectile motion
- projectile range
- projection vector
- resolving a vector
- resultant vector
- scalar
- skew
- system of linear equations
- two-/three-dimensional vector

Literacy Skills

Written

- using technical /procedural vocabulary
- using conventions (symbols, abbreviations) e.g.

- $\forall$, $\in$, RTP, QED, LHS, RHS, N (set), z^+ (set), $\overrightarrow{AB}$, ç, **d**
- $|\overrightarrow{AB}|$, |ç| and |**d**|, $\hat{\imath}, \hat{\jmath}, \hat{k}$, ç = $5\hat{\imath} + 3\hat{\jmath} - 2\hat{k}$- $A(2,3,1)$, $\overrightarrow{OA} = (5,3,-2)$, $x-y, y-z, x-z$ planes, MMAA, DMSA, Re(z), Im(z)
- $k \in R$, $C_1 : \begin{cases} x = k+1 \\ y = k^2 \end{cases}$, x_c is the x-coordinate of point C
- $\overrightarrow{AP} . (\overrightarrow{AB} \times \overrightarrow{AC}) = 0, \dot{r}(t)$ and $\ddot{r}(t)$
- $x(t) = sin(t), y(t) = cos(2t), 0 \leq t \leq \frac{\pi}{2}$
- $R_1 - \frac{1}{2}R_2 \rightarrow R_2$, $\left(\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 1 & -2 & 2 & 4 \\ 1 & 2 & -1 & 2 \end{array}\right)$, $i\ where\ i^2 = -1$
- $a + bi$ (complex form), $|z|$, - arg(z), mod(z), cis, $\bar{z}$

Oral

- articulating technical and procedural vocabulary, proving results
- articulating the different ways vectors can be expressed in two dimensions, locating complex conjugates in the complex plane, linear transformation in the complex plane
  - an algebraic method, - an analytic method
  - factorising the polynomial,
  - expressing as a product of linear factors
  - solving the complex polynomial equation

21st Century Skill/s

Critical thinking
- reasoning
- reflecting
- problem-solving
- decision-making
- analytical thinking
- intellectual flexibility

Communication
- using language, symbols and texts
- creative thinking
- seeing new links

Collaboration and teamwork
- relating to others
- participating and contributing

Creative thinking
- making connections
- generating and applying new ideas

ICT skills
- accessing and analysing information
- being productive users of technology

<table>
<tr>
<td rowspan="2">• orthogonal
• parabola
• parallel
• parametric form of an equation
• parametric equation of a line
• perpendicular
• polar form
• position vector</td>
<td rowspan="2">• unique solution
• unit vector
• vector equation of a line
• vector equation of a plane
• vector form of an equation</td>
<td>Visual
▪ using graphs, tables, grids, applets, spreadsheet patterns, diagrams, scale pictures, Cartesian planes, vector diagrams modelling real-life situations, two- and three-dimensional perspectives
▪ using Cartesian plane, Argand plane, number line, polar grid</td>
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<td>Numeracy Skills
▪ calculating with whole numbers
▪ recognising and using patterns and relationships
▪ showing capacity to use mathematical knowledge in a range of contexts
▪ using digital tools, diagrams, spreadsheets, etc.
▪ making informed decisions based on the expected returns in game theory
▪ calculating with irrational numbers
▪ working with directions and bearings
▪ recognising and using patterns and relationships
▪ making decisions and judgments with critical orientation</td>
<td>Cognitive Verbs
Retrieval and Comprehension: review, use, summarise, write, identify, recognise, demonstrate, select, recall, sketch, show, consolidate, describe, comprehend, calculate, explain

Analysis: determine, contrast, apply. consider, revise

Knowledge Utilisation: design, debate, discuss, prove, investigate, propose, express, argue, link, solve, develop, explore, make decisions, realise, devise, justify, construct, establish, challenge, evaluate, individually, show, conduct, explain, generate</td>
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TEACHING AND LEARNING PLAN:

Hours/Weeks	Unit Objectives	Subject Matter	Learning Experiences [reflecting DQ 3, 4, 5 and 6]	Possible Resources
Unit 3 *Week 1* Term 4 (Yr11) Week 5 3 Hours	1,2,3,4,5,6	**Proof by Mathematical Induction** **Mathematical induction (7 hours)** • understand the nature of inductive proof including the 'initial statement' and inductive step • prove results for sums for any positive integer n. • prove divisibility results for any positive integer n.	Refer to QCAA TLAP Unit 3 Specialist Mathematics	Textbook Maths Quest Specialist Mathematics Units 3&4 (Jacaranda) Digital version is also available
Unit 3 *Weeks 2 – 9* Term 4 (yr11) Weeks 6-7 + Term 1 (yr12) Weeks 1 - 6 24 Hours	1,2,3,4,5,6	**Vectors & Matrices** **The algebra of vectors in three dimensions (4 hours)** • review the concepts of vectors from Unit 1 and extend to three dimensions by introducing the unit vector $\hat{k}$ and the altitude φ • prove geometric results (review from the topic Geometric proofs using vectors) in the plane and construct simple proofs in three dimensions. **Vector and Cartesian equations** • introduce Cartesian coordinates for three-dimensional space, including plotting points and the equations of spheres • use vector equations of curves in two or three dimensions involving a parameter, and determine a 'corresponding' Cartesian equation in the two-dimensional case • determine a vector, parametric and Cartesian equation of a straight line and straight-line segment given the position of two points, or equivalent information, in both two and three dimensions • examine the position of two particles, each described as a vector function of time, and determine if their paths cross or if the particles meet • define and use the vector (cross) product to determine a vector normal to a given plane • use vector methods in applications, including areas of shapes and determining vector and Cartesian equations of a plane and of regions in a plane. **Systems of linear equations (6 hours)** • recognise the general form of a system of linear equations in several variables and use Gaussian techniques of elimination to solve a system of linear equations • solve systems of linear equations using matrix algebra • examine the three cases for solutions of systems of equations — a unique solution, no solution and infinitely many solutions — and the geometric interpretation of a solution of a system of equations with three variables. **Applications of matrices (7 hours)** • model real-life situations using matrices, including Dominance and Leslie • investigate how matrices have been applied in other real-life situations, e.g. Leontief, Markov, area, cryptology, eigenvectors and eigenvalues. **Note:** The external examination may assess only Dominance and Leslie matrices.	Refer to QCAA TLAP Unit 3 Specialist Mathematics	

		Vector calculus (5 hours) • consider position of vectors as a function of time • derive the Cartesian equation of a path given as a vector equation in two dimensions, including circles, ellipses and hyperbolas • differentiate and integrate a vector function with respect to time • determine equations of motion of a particle travelling in a straight line with both constant and variable acceleration • apply vector calculus to motion in a plane, including projectile and circular motion.		
Unit 3 *Weeks 10 - 13* Term 1 Weeks 7 -10 12 hour	1,2,3,4,5,6	**Complex Numbers 2** **Cartesian forms (4 hours)** • review real and imaginary parts $Re(z)$ and $Im(z)$ of a complex number z • review Cartesian form • review complex arithmetic using Cartesian form. **Complex arithmetic using polar form (3 hours)** • prove the identities involving modulus and argument • prove and use De Moivre's theorem for integral powers. **The complex plane (the Argand plane) (2 hours)** • identify subsets of the complex plane determined by straight lines and circles. **Roots of complex numbers** • determine and examine the nth roots of unity and their location on the unit circle • determine and examine the nth roots of complex numbers and their location in the complex plane. **Factorisation of polynomials** • prove and apply the factor theorem and the remainder theorem for polynomials • consider conjugate roots for polynomials with real coefficients • solve polynomial equations to order 4.	Refer to QCAA TLAP Unit 3 Specialist Mathematics	